New Exact Travelling Wave Solutions of Toda Lattice Equations By Using New Mathematical Model

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Abstract— In this paper, we introduce a new mathematical model to obtain new exact travelling wave solutions for Toda lattice equations, we obtain the solutions and the results are presented graphically. By comparison our new solutions by the other results we found that our solutions are new and not be found before, also these solutions take the form $u_n(t)=\varpi\tanh(nd+\varpi t)+\varpi\dfrac{nd+\varpi t}{\sinh^2 d}-t, n\in\mathbb{Z}$..

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Index Terms— Amplitude-frequency formulation, differential-difference equation, solitary solution, period.

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1 Introduction

Discrete nonlinear lattices have been the focus of considerable attention in various branches of science. Many differential equations on the nano scales are invalid, but the problems arising can be well modeled by differential-difference equations [1-5]. As is well known, there are many physically interesting problems such as charge fluctuations in net work, ladder type electric circuits, phenomena in crystals, molecular chains, which all can be modelled by differential-difference equations [6-10].

Recently many analytical methods were proposed to solve differential-difference equations, such as the exp-function method[11-13], the ancient Chinese algorithm [14], the variational iteration method[15,16,17], the homotopy perturbation method[18,19,20]. The mixed function method [21]. A complete review on various analytical methods is available in Refs [21].

In this paper we introduce a new mathematical model to obtain new exact travelling wave solutions of Toda lattice equations, the results are presented graphically. By comparison our new solutions by the other results we found that our solutions are new and not be found before.

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2 Amplitude-Frequency Formulation for Nonlinear Oscillators

Consider a generalized nonlinear oscillator in the form

$$u''+f(u)=0,\qquad u(0)=A,\qquad u'(0)=0 \tag{1}$$

We use a trial functions

$$u=A\cos\omega_1 t \tag{2}$$

which is, the solution of the following linear oscillator equations

$$u''+\omega_1^2 u=0\,, \tag{3}$$

where ω_1 is trial frequency which can be chosen freely, for example, we can set $\omega_1=2$ or $\omega_1=\omega$, where ω is assumed to the frequency of the nonlinear oscillator.

Substituting Eq.(2) into, Eq.(1), we obtain the following residual

$$R(t)=-A\omega_1^2\cos\omega_1 t+f(A\cos\omega_1 t) \tag{4}$$

an amplitude-frequency formulation for nonlinear oscillators can be found from the equation was proposed [18]

$$R(t)=0 \tag{5}$$

3 Solitary-solution Formulation for Differential Difference Equations

Suppose the differential-difference equation we discuss

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in this paper is in the following nonlinear polynomial form:

$$\frac{du_n(t)}{dt} = f(u_{n-1}, u_n, u_{n+1}) \qquad (6)$$

where $u_n = u(n,t)$ is a dependent variable; t is a continuous variable; $n,\ p_i \in \mathbb{Z}$.

We choose a trial-function in the forms:

$$u_n(n,t) = f(\xi_n + \omega\, t) \qquad (7)$$

where $\xi_n = nd + \xi_0$, ξ_0 is arbitrary, f is known functions. If a periodic solution is searched for, f must be periodic function; If a solitary solution is solved, f must be of solitary structure. In this paper a bell solitary solution of a differential-difference equation is considered, and trial-functions are chosen as follows

$$u_n(n,t) = \frac{A\left(e^{nd+\xi_0+\omega t} - e^{-(nd+\xi_0+\omega t)}\right)}{e^{nd+\xi_0+\omega t} + e^{-(nd+\xi_0+\omega t)}} + B\left(nd + \xi_0 + \omega t\right), \qquad (8)$$

ξ_0 can be taken to be zero without loss of generality because the system (6) is autonomous.

By replacing n by $n-1$ and n by $n-2$, we have

$$u_{n-1}(n,t) = \frac{A\left(e^{(n-1)d+\omega t} - e^{-((n-1)d+\omega t)}\right)}{e^{(n-1)d+\omega t} + e^{-((n-1)d+\omega t)}} + B\left(\left(n-1\right)d + \omega t\right), \qquad (9)$$

$$u_{n+1}(n,t) = \frac{A\left(e^{(n+1)d+\omega t} - e^{-((n+1)d+\omega t)}\right)}{e^{(n+1)d+\omega t} + e^{-((n+1)d+\omega t)}} + B\left(\left(n+1\right)d + \omega t\right). \qquad (10)$$

Solution procedure

Step 1: Define residual function

$$\tilde{R}(t) = \frac{du_n(t)}{dt} - f\left(u_{n-1}, u_n, u_{n+1}\right) \qquad (11)$$

Substituting Eqs. (8)~(10) into Eq. (6), we can obtain , respectively, the residual function $\tilde{R}(t)$.

$$\tilde{R}(t) = \frac{du_n(t)}{dt} - f\left(u_{n-1}, u_n, u_{n+1}\right)$$

Step 2: Solitary-solution formulation is constructed as follows

$$\tilde{R}(t) = 0 \qquad (12)$$

Step 3: Combining the coefficients of e^{ξ_n} in Eq.(12), and setting them to be zero, we can solve the algebraic equations to find the values of ω, A and B. Finally an explicit solution is obtained.

4 Application in Discrete Toda Lattice Equation

Consider the following Toda difference equation

$$\frac{d}{dt^2}u_n(t) = \left(\frac{d}{dt}u_n(t) + 1\right)\left(u_{n+1}(t) - 2u_n(t) + u_{n-1}(t)\right),$$

where $n \in \mathbb{Z}, t \in \mathbb{R}$. In order to simplify the equation , we set $v_n(t) = \left(u_n(t) + t\right)$. Then we obtain

$$\frac{d}{dt^2}v_n(t) = \frac{d}{dt}v_n(t)\left(v_{n+1}(t) - 2v_n(t) + v_{n-1}(t)\right). \qquad (13)$$

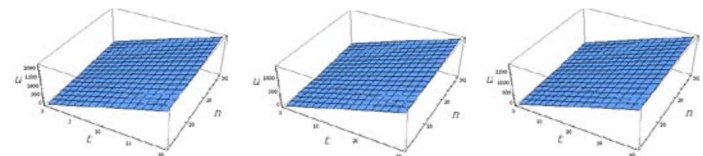
We try to find a solution (traveling wave solution) by using our new algorithm,

Substituting Eqs. (8)~(10) into Eq. (13), and after simple calculation we obtain the more interesting solution

$$A = \omega, B = \frac{\omega}{\sinh^2 d}.$$

Therefore, we found the solution

$$u_n(t) = \omega \tanh\left(nd + \omega t\right) + \omega \frac{nd + \omega t}{\sinh^2 d} - t, n \in \mathbb{Z}$$



omega=2, d=0.2	omega=2, d=0.3	omega=2, d=0.4

5 Conclusion

In this paper, though there are many analytical methods, such as the exp-function method, the variational iteration method, the homotopy perturbation method, for differential-difference equations, this paper suggests an effective and simple approach to such problems using the new algorithm, and the simple formulation can be used routinely by followers to various differential-difference equations.

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